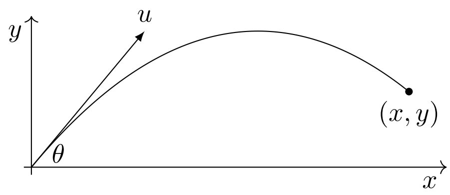
4301. Carry out the following integral:

$$I = \int \left(\frac{\ln x}{x} \right)^2 dx.$$

4302. A projectile is launched from an origin, at fixed speed u, at some angle θ above the horizontal.



You are given that there is exactly one angle of projection θ which will hit point (x, y). Show that

$$y = \frac{u^2}{2g} - \frac{gx^2}{2u^2}.$$

4303. Prove that no polynomial of odd degree can be concave everywhere.

4304. A computer generates random numbers $X_1, X_2, ...,$ each distributed independently as $X \sim N(0, 1)$. Show that, for any constant $k \in [0, \infty)$,

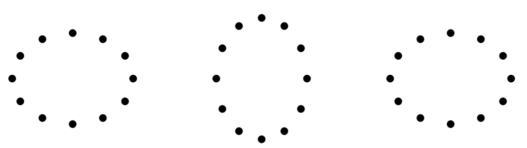
$$\lim_{n \to \infty} \mathbb{P} \left(k < \sum_{i=1}^{n} X_i \right) = \frac{1}{2}.$$

4305. Show that the sum of the integers from 1 to ab which are not divisible by a is $\frac{1}{2}a(a-1)b^2$.

4306. Show that, if $\lim_{n \to \infty} b_n = k$ for some $k \in \mathbb{R}$, then

$$\sum_{i=1}^{\infty} (b_i - b_{i+1}) = b_1 - k.$$

4307. The effect of a passing *gravitational wave* is often described as transforming a circle of test particles into an undulating ellipse.



A simple model for this undulation is

$$x = \text{f}(t) \cos s, \quad y = \text{g}(t) \sin s,$$

where f and g are functions, t is time and $s \in [0, 2\pi)$ is a parameter.

(a) Eliminate the parameter s to find the time-varying Cartesian equation of the ellipse.

(b) General relativity predicts that the area of the ellipse must be constant. Give the relationship between $\text{f}(t)$ and $\text{g}(t)$.

4308. A function, with codomain $\mathbb{R}$, has instruction

$$\text{f}(x) = \sum_{i=1}^{\infty} (1 - 2x)^i.$$

(a) Determine the largest real domain over which this function is well defined.

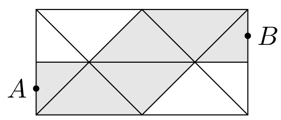
(b) Find the range of f over this domain.

4309. Prove that $y = x^{2p} + x^{2q+1}$, where $p, q \in \mathbb{N}$, has neither reflective nor rotational symmetry.

4310. Four identical cannonballs of mass m are stacked together on horizontal ground, forming a pyramid. Each cannonball is tangent to all of the others. Show that the reaction force exerted on the upper cannonball by each of the other three is

$$R = \frac{mg}{\sqrt{6}}.$$

4311. In the schematic map below, six of the ten regions are shaded. The six regions are contiguous, so that there is an unbroken path from A to B through shaded regions.



Six regions are now shaded at random. Find the probability that points A and B end up connected by contiguous shaded regions.

(Regions are contiguous if they share a border, but not if they only share a vertex.)

4312. Simplify $I = \int e^{\tan x} \, dx + \int e^{\tan x} \tan^2 x \, dx.$

4313. Point P on the unit circle is rotated anticlockwise around the origin of an (x, y) plane, by angle 2θ.

(a) Show that the distance between the initial and final positions of P is $\sqrt{2 - 2\cos 2\theta}$.

(b) Write this in terms of $\sin \theta$.

(c) P undergoes n such rotations, and completes a full circle. Express n in terms of θ.

(d) Hence, prove the small-angle approximation $\sin \theta \approx \theta$, by showing that

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1.$$

4314. Using integration by parts, show that

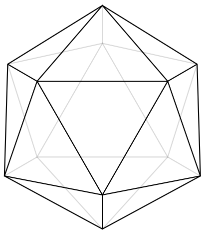
$$\int (\ln x)^2 \, dx = x(\ln x)^2 - 2x \ln x + 2x + c.$$

4315. Prove that the derivative of $\sec x$ with respect to $\tan x$ is $\operatorname{cosec} x$.

4316. Determine the largest real domain over which it is possible to define the function

$$\mathrm{f} : x \mapsto \frac{1}{\sqrt{x^4 + x^2 - 6}}.$$

4317. An icosahedron has 20 equilateral triangular faces. Five faces meet at each vertex.



A design for an icosahedral die has the numbers 1 to 20 printed on these faces at random. Find the probability that none of the numbers 1 to 10 share an edge.

4318. The hyperbola $x^2 - y^2 - k = 0$ and the parabola $x^2 + ky = 0$ have precisely two distinct points of intersection. Show that $k \in (-\infty, 0) \cup \{4\}$.

4319. A normal is drawn to the curve $y = \cos x$ at $x = a$. Show that, for a close to zero, the equation of the normal is approximately $2ay - 2x + a^3 = 0$.

4320. *Hooke's Law* states that the force F exerted by a stretched spring is proportional to the extension x of the spring from its natural length.

In this question, extension x m and acceleration a ms^{-2} are 1D vectors, which have the same sense of positive direction as each other.

A mass, resting on a smooth horizontal surface, is attached to one end of a horizontal spring, the other end of which is fixed. The spring is stretched, and the mass then released. Resistance forces are neglected.

(a) Show that, for some positive constant q, the acceleration a is given by $a = -qx$.

(b) Verify that, for any constants $A, B \in \mathbb{R}$, the following general solution satisfies the above differential equation:

$$x = A \sin \sqrt{q}t + B \cos \sqrt{q}t.$$

(c) Describe the long-term kinematic behaviour of the system, according to this model.

4321. A tangent is drawn to $y = x^3$ at the point with x coordinate p. Prove that zero is the only value of p for which the tangent line does not re-intersect the curve elsewhere.

4322. In a statistical procedure, variables X and Y are modelled with distributions

$$X \sim N(-1, 1),$$
$$Y \sim N(1, 1).$$

Running a small number of trials suggests that

$$\mathbb{P}\left(X > 0 \text{ and } Y < 0\right) \approx \tfrac{1}{40}.$$

Determine if this is consistent with the assumption that X and Y are independent.

4323. The function f is defined, over $\mathbb{R}$, by

$$\mathrm{f}(x) = \cos x(x - \sin x).$$

You are given that $\mathrm{f}(x)$ is stationary at exactly one x value in the domain $\left(0, \frac{\pi}{2}\right)$. Two claims are made about the function:

(a) "For small x, $\mathrm{f}(x) \approx 0$."

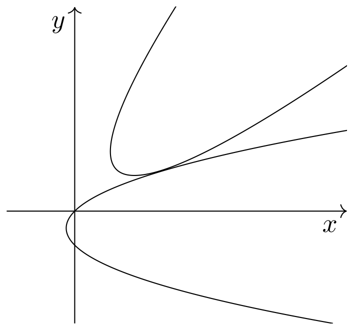
(b) "Over the domain $[\pi/2, \pi/2]$, $\mathrm{f}(x) \approx 0$."

Explain whether these claims are true.

4324. The diagram shows the curves

$$x = y^2 + y,$$
$$x + y = (x - y)^2 + k.$$

The curves are tangent.



Find the value of the constant k.

4325. Size P of a population is modelled with the DE

$$2e^{-t}\frac{dP}{dt} - \frac{t}{P} = 0.$$

Initially, $P = 100$.

Find, to the nearest whole number, the value of P predicted by the model at $t = 10$.

4326. State, giving a reason, which of the implications $\Longrightarrow$, $\Longleftarrow$, $\Longleftrightarrow$ links the following statements concerning polynomial functions f and g:

① $\mathrm{f}(a) = \mathrm{g}(a)$,
② $\mathrm{f}(x) - \mathrm{g}(x)$ has a factor of $(x - a)$.

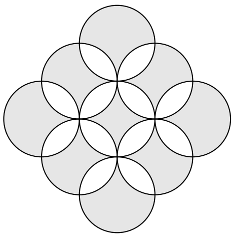
4327. In a fairground game, a prize oscillates above a small chute, held in a pair of jaws. The player chooses the point of release, hoping to drop the prize into the chute. The mouth of the chute is taken to be at the origin of a vertical (x, y) plane. The prize oscillates in this plane according to the following equations:

$$x = \cos t,$$
$$y = 1.$$

Show that, if the prize is to reach the mouth of the chute, then the time of release t_0 must satisfy

$$\tan t_0 = \pm\sqrt{\frac{g}{2}}.$$

4328. The diagram below shows nine identical circles of radius r arranged in a symmetrical pattern.



Show that the pattern has total area $3r^2(\pi + 4)$.

4329. The *hyperbolic sine* function sinh is defined as

$$\sinh x = \frac{e^x - e^{-x}}{2}.$$

(a) The derivative of hyperbolic sine is hyperbolic cosine. Give, in a similar form, the definition of the cosh function.
(b) Show that $y = \sinh x$ and $y = \cosh x$ have one stationary point between them.
(c) Sketch $y = \sinh x$ and $y = \cosh x$ on the same set of axes.

4330. A statistical model uses distribution functions of the following form, for $n \in \mathbb{N}$:

$$\mathrm{f}_n(x) = kx(1-x)^n.$$

These functions are defined over the domain $[0, 1]$, and k is fixed by the requirement that the area beneath $y = \mathrm{f}_n(x)$ is 1.

(a) Show that, when $n = 2$, $k = 12$.
(b) Find a general formula for k in terms of n.

4331. The outcome of an experiment is modelled as X, where $X \sim B(5, 1/3)$. Find $\mathbb{P}(X \leq 4 \mid X > 2)$.

4332. Find the largest possible real domain over which $\mathrm{f}(x) = \operatorname{arcsec}(2x + 1)$ may be defined, and the range over this domain.

4333. A basketball is aimed at a hoop. It is thrown at 8 ms^{-1}, from 2 m away horizontally and 1 m below the hoop. Air resistance is modelled as negligible.

(a) Show that the angle of projection must satisfy

$$49\tan^2\theta - 320\tan\theta + 209 = 0.$$

(b) Determine the possible angles of projection to reach the hoop.
(c) Show that only one of the possible trajectories will allow the basketball to drop down through the hoop.

4334. A straight line $y = mx + c$ intersects a cubic graph $y = \mathrm{g}(x)$ at three distinct points A, B, C. The central point B is the cubic's point of inflection.

Show that the coordinates of A, B, C are in AP.

4335. Show that $\displaystyle\int_0^{\pi} (x^2 + 1)\cos 4x \, dx = \frac{\pi}{8}$.

4336. A curve has equation

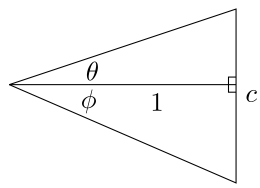
$$x^2 + 2x + y^3 = 63 + xy.$$

Determine the coordinates of any SPs.

4337. Sketch the graph $y = \dfrac{|x+1|}{x+1} + \dfrac{|x-1|}{x-1}$.

4338. In the triangle below, use the cosine rule to prove the compound-angle formula

$$\cos(\theta + \phi) \equiv \cos\theta\cos\phi - \sin\theta\sin\phi.$$



4339. A coin is tossed six times. Find the probability that there is a run of at least four consecutive tosses giving the same result.

4340. A particle is moving in 2D. Its position vector at time t given, for some constant $k \in \mathbb{N}$, by

$$\mathbf{r}_t = \begin{pmatrix} \sin(2k+1)t \\ \cos(2k+3)t \end{pmatrix}.$$

Show that the particle never comes to rest.

4341. Solve the following equation:

$$\frac{x^4 + 3x^2 + 2}{x^2 + 3x + 2} = \frac{x^4 - 3x^2 + 2}{x^2 - 3x + 2}.$$

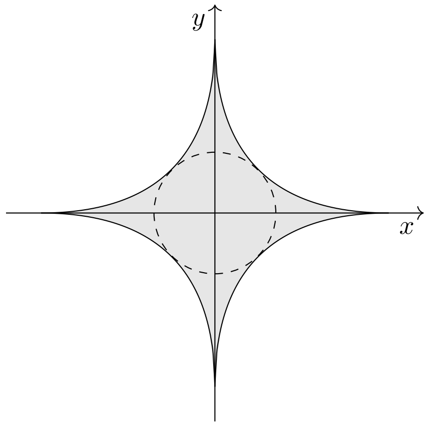
4342. Two students are discussing approaches to

$$\int x\sqrt{2x-3}\,dx.$$

One says "It's a product, so we should use parts." The other says "No, there's a composition, so we should use substitution, with $u = 2x - 3$."

Resolve the issue.

4343. The graph $x^{\frac{1}{2n}} + y^{\frac{1}{2n}} = 1$, where $n \in \mathbb{N}$ is a constant, is drawn in the positive quadrant. The graph is then rotated around O to produce a shape with symmetry order 4.



The largest possible circle is drawn inside the shape. Its area is defined as A_n. Show that

$$\lim_{n\to\infty} A_n = 0.$$

4344. The partial sums of the *harmonic series* are

$$S_n = \sum_{r=1}^{n} \frac{1}{r}.$$

(a) By considering the summand as the area of a rectangle of width 1, show that

$$S_n > \int_1^{n+1} \frac{1}{x}\,dx.$$

(b) Hence, prove that the series diverges.

4345. Giving all roots in $[0, 2\pi)$, solve the equation

$$\left(|\sin x| + \tfrac{1}{2}\right)\left(|\sin x| - \tfrac{1}{2}\right) = 0.$$

4346. Two sets of data have medians $m_1 < m_2$ and IQRs $I_1 < I_2$. When combined into one larger set, they have median m_c and IQR I_c.

State, with a reason, whether each of the following inequalities is necessarily true:

(a) $m_1 < m_c < m_2$,

(b) $I_1 < I_c < I_1$.

4347. The error E in the approximation $\sin\theta \approx \theta$, for some small angles in radians, is given below.

θ	0.2	0.4	0.6
E	0.0013	0.0106	0.0354

It is proposed that, for small angles, this error is proportional to the cube of θ, giving $E \approx k\theta^3$, for some $k \in \mathbb{R}$.

(a) Show that, if this cubic relationship holds, then $\ln E$ and $\ln\theta$ should be related linearly.

(b) Plot $\ln E$ against $\ln\theta$ on a graph, and obtain a line of best fit.

(c) Verify that a cubic approximation is valid, and show that $k \approx \frac{1}{6}$.

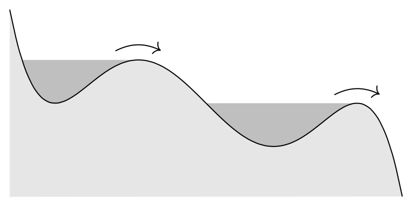
4348. Prove the *remainder theorem*, which states that, if a polynomial $\mathrm{p}(x)$ is divided by $(x - \alpha)$, then the remainder is given by $\mathrm{p}(\alpha)$.

4349. In a regular tetrahedron $ABCD$, every face is an equilateral triangle. Prove that the angle between the face ABC and the edge AD is $\arctan\sqrt{2}$.

4350. A sample size N is taken from a large population normally distributed around mean μ. Show that, for any constant $a \in (0, \infty)$,

$$\lim_{N\to\infty} \mathbb{P}\left(|\bar{X} - \mu| < a\right) = 1.$$

4351. A cascading stream, with two pools, is modelled, in profile, by the equation $z = -x^5 + 2x^3 - x$, with units of metres. In the horizontal y dimension not shown in the profile, the stream is 3 metres wide.



(a) Show that the lower pool has volume 0.5 m^3.

(b) Find, to 3sf, the volume of the upper pool.

4352. Show that the three lines $y = -x$, $y = \left(2 \pm \sqrt{3}\right)x$ divide the unit circle into six regions of equal area.

4353. State, with a reason, whether $y = x^{100}$ intersects the following curves:

(a) $y = x^{98} + 1$,

(b) $y = x^{99} + 1$,

(c) $y = x^{100} + 1$,

(d) $y = x^{101} + 1$,

(e) $y = x^{102} + 1$.

4354. By simplifying the integrand, show that

$$\int_0^1 \tan(\arcsin t)\,dt = 1.$$

4355. Show that the area of the region for which $x \geq |y|$ and $y \geq |4x - 10| - 5$ both hold is $\frac{40}{3}$.

4356. An epidemiologist is working with two models for the **rate** r at which new infections occur, from an outbreak at $t = 0$. The variable t is the number of days after outbreak.

① The established model is cubic: $r = t^3$.
② The new model is exponential: $r = e^{kt} - 1$.

(a) Verify that the models coincide at $t = 0$.
(b) The value of k is chosen so that the models also coincide at $t = 10$. Show that $k \approx \ln 2$.
(c) In this part, use the exact value $k = \ln 2$. The models are now used to predict **total** number of new infections for $t \in [0, 10]$. Show that the new model predicts approximately 41% fewer new infections.

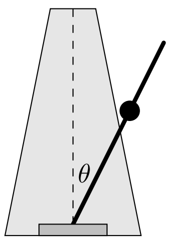
4357. Consider the graph $y = \left(\dfrac{x^2 + 4}{2x}\right)^4$.

(a) Show that y is never zero.
(b) Find the two stationary points of the curve.
(c) Hence, sketch the curve.

4358. Derive the Newton-Raphson formula

$$x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}.$$

4359. The moving part of a metronome consists of a light metal bar, length 10 cm, hinged at its lower end, to which is attached a bob of mass 200 g, whose position on the bar may be varied.



(a) To begin with, the bar is in equilibrium at an angle of $\theta = 30°$ to the vertical. The bob is at the top end. Determine the moment being applied by the hinge mechanism.
(b) The bob is repositioned halfway down the bar; the bar swings towards vertical. At $\theta = 30°$, the acceleration of the bob is 2.5 ms^{-2}. Find the magnitude of the force exerted by the bar on the bob at this instant.

4360. Three couples sit down at random around a round table. Find the probability that no one is sitting next to their partner.

4361. To differentiate $y = \tan x$ from first principles, the following limit is set up

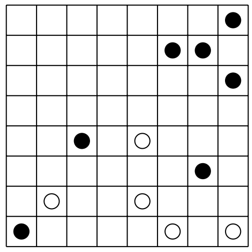
$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\tan(x + h) - \tan(x)}{h}.$$

Evaluate the limit to prove that $\dfrac{dy}{dx} = \sec^2 x$.

4362. Triangle OAB has vertices $(0, 0)$, $(18, 6)$, $(8, 16)$. Determine the coordinates of the point P within the triangle for which the following quantity is maximised:

$$\min\big(|OP|, |AP|, |BP|\big).$$

4363. A set of r_1 identical black and r_2 identical white counters are to be placed on a board of n squares, such that no more than one counter is on any one square.



Prove that the number of ways of doing this is

$$\frac{n!}{r_1!\, r_2!\, (n - r_1 - r_2)!}.$$

4364. Solve the simultaneous equations

$$3x^2y^2 + 2xy - 1 = 0,$$
$$x + 2y = 1.$$

4365. "The hyperspace diagonal (longest diagonal) of a five-dimensional unit hypercube has length $\sqrt{5}$."

True or false?

4366. Sketch the implicit relation $\log_x y + \log_y x = 4$.

4367. A real number k is generated randomly, from a uniform distribution, on the interval $(0, 1)$. Find the probability that

(a) $k^2 < k$,
(b) $k^2 < k - \frac{1}{4}$,
(c) $k^2 < k - \frac{3}{16}$,

4368. Determine the area of the region defined by

$$0 < x^3 < y < x^2 + 12x.$$

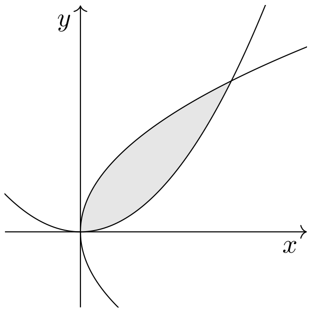
4369. Functions f and g have the same second derivative. Prove that the equation $\mathrm{f}(x) = \mathrm{g}(x)$ is satisfied by either zero, one or infinitely many x values.

4370. The *dot product* is defined, in terms of the angle θ between two vectors, by $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$. Verify geometrically that

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 17.$$

4371. Sketch the graph $y = \dfrac{1}{x^2 - 2|x|}$.

4372. The graphs $4y = x^2$ and $4x = y^2$ enclose a region of the plane.



Show that the largest circle that will fit inside this region has diameter $\sqrt{2}$.

4373. A quadratic equation Q is given as

$$ax^2 + bx + c = 0.$$

The coefficients $a < b < c$ are three consecutive non-zero integers. Show that Q has no real roots.

4374. A particle is projected from ground level at speed u, at an angle θ above the horizontal, and attains a maximum height h and a range d.

(a) Find $\sin^2\theta$ in terms of u, g and h.

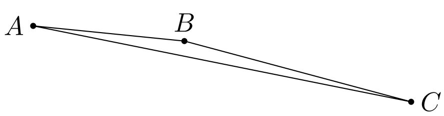
(b) Hence, show that

$$d = \frac{\sqrt{8gh(u^2 - 2gh)}}{g}.$$

4375. A hand of five cards is dealt from a standard deck. Find the probability that the hand is a full house, i.e. three of one number and two of another.

4376. Sketch the graph $y = \operatorname{arccot}(x^2)$.

4377. Consider a thin triangle ABC, in which angle C, defined in radians, is small.



(a) Show that $c^2 \approx (a-b)^2 + a^2b^2\theta^2$.

(b) Interpret this result as $\theta \to 0$.

4378. Limit L is defined for non-zero $a, b, n, x \in \mathbb{R}$:

$$L = \lim_{h \to 0} \frac{(ax+h)^n - (ax-h)^n}{(bx+h)^n - (bx-h)^n}.$$

Show that $L = \dfrac{a^{n-1}}{b^{n-1}}$.

4379. Four distinct edges are chosen at random from among those of a regular n-gon, where $n = 4k$ for some $k \in \mathbb{N}$. Show that the probability of these edges lying on the edges of a square is

$$\frac{6}{n^3 - 6n^2 + 11n - 6}.$$

4380. Shade the region of the (x, y) plane defined by the simultaneous inequalities

$$\begin{aligned} x + y &\geq 1, \\ x^2 + y^2 &\geq 1, \\ x^3 + y^3 &\leq 1. \end{aligned}$$

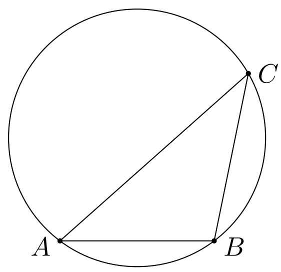
4381. A uniform ladder of length l and mass m is resting in equilibrium against a smooth wall, with its foot a distance d from the base of the wall. A person of mass $6m$ is standing on a rung $\frac{1}{4}l$ from the foot of the ladder. The ground is rough, coefficient of friction μ. The ladder is in equilibrium. Show that

$$\mu \geq \frac{2d}{7\sqrt{l^2 - d^2}}.$$

4382. A normal is drawn to the parabola $y = x^2$ at $x = p$. The normal crosses the parabola again at $x = q$. Prove that $q^2 - p^2 > 1$.

4383. Solve the equation $2\sqrt[3]{a^2} + \sqrt[3]{a} - 1 = 0$.

4384. A circle, radius r, has a triangle drawn inside it, with vertices A, B, C on the circumference.



(a) Assume that A and B are fixed. Explain why the triangle's area is maximised when C is moved to form an isosceles triangle.

(b) Let $\triangle ABC$ be isosceles, with $|AB| = 2x$ as the base and $|AC| = |BC|$ as equal slant heights. Show that the area can be expressed as

$$A_\triangle = x\left(r + \sqrt{r^2 - x^2}\right).$$

(c) Hence, or otherwise, prove that the maximal area for an inscribed triangle occurs when the triangle is equilateral.

4385. The function $\mathrm{f}(x) = ax^5 + bx^3$ is invertible over the entire real number line. On a set of (a, b) axes, shade the region(s) representing all possible values of the constants a and b.

4386. The cubic equation $8x^3 - 36x^2 + 46x - k = 0$, for some constant $k \in \mathbb{R}$, has three roots in arithmetic progression. Determine k and solve the equation.

4387. The *integrating factor* method for differential equations involves writing an expression as $\frac{d}{dx}(uv)$, where u and v depend on x. A DE is given as

$$\frac{dy}{dx} + y \tan x = 0.$$

For this equation, the integrating factor I is chosen such that

$$\frac{d}{dx}(\ln I) = \tan x.$$

(a) Verify that $I = \sec x$ is an integrating factor for this differential equation.

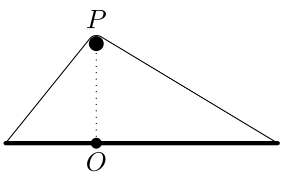
(b) The DE is multiplied by I. Show that the LHS may be written as the derivative of a product.

(c) Hence, show that the general solution of the differential equation is $y = A \cos x$.

4388. A hand of four cards is dealt from a standard deck. Determine the probability that the hand contains exactly two suits.

4389. Prove that the curves $y = \sin x + p$ and $x = \sin y + q$ intersect exactly once, regardless of the value of the constants p and q.

4390. A rigid, uniform bar of length $3d$ and weight W is hinged freely at O, a point dividing its length in the ratio $1 : 2$. A light, inextensible rope is passed over a smooth peg a distance $\sqrt{3}d$ above O at P, and is attached to the ends of the bar such that the rope is taut.



Show that, for the system to remain in equilibrium, the rope must be tightened to a tension of

$$T = \frac{7\sqrt{3} + 4\sqrt{21}}{27} W.$$

4391. Show carefully that no values of x satisfy

$$x^2 < \sin x < x^3.$$

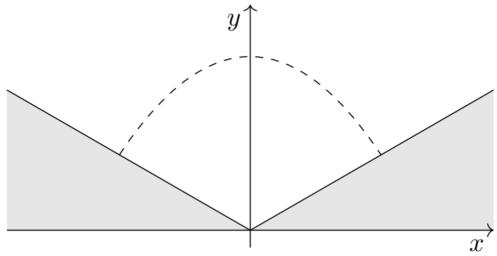
4392. Show that $\displaystyle\int_0^\infty |x + 1| \, e^{-x} \, dx = 2.$

4393. Circles C_n, for $n \in \mathbb{N}$, are placed such that

- each circle is centred on the x axis,
- C_{n+1} is tangent to C_n,
- the diameters of the circles are in GP,
- no circle lies inside another,
- the total x-length of the sequence is 100,
- the combined area of C_1 and C_2 is $\frac{181}{4}\pi$.

Find the diameter of C_2.

4394. A projectile is bouncing periodically between two surfaces defined by the equation $\sqrt{3}y = |x|$. The projectile's path is a single parabola. At $x = 0$, it has speed 0.7 ms^{-1}.



(a) Write down the gradients of the parabola at the points at which the projectile bounces.

(b) Find y when $x = 0$.

4395. A general pair of simultaneous equations in x and y are given below, for constants a, b, c, d, p, q:

$$\begin{aligned} ax + by &= p \\ cx + dy &= q. \end{aligned}$$

(a) Show that these equations always have at least one solution, provided that $a/c \neq b/d$.

(b) If, in fact, $a/c = b/d$, determine the condition on p and q for these equations to have infinitely many solutions.

4396. Find the maximum value of the quantity $2x + y$ on the circle $x^2 + y^2 = 1$.

4397. Prove the following identity:

$$\sin\left(\tfrac{\pi}{4} + \tfrac{x}{2}\right) \operatorname{cosec}\left(\tfrac{\pi}{4} - \tfrac{x}{2}\right) \equiv \sec x + \tan x.$$

4398. A curve has parametric equations, for $t \in [0, \pi/2)$:

$$\begin{aligned} x &= \sin^2 t, \\ y &= \sin t. \end{aligned}$$

Determine the area of the region enclosed by the curve, the x axis and the line $x = 1$.

4399. A fair die is rolled three times, and each score turns out to be higher than the previous one. Find the probability that the last roll was a six.

4400. A particle moves along an x axis with velocity given by $v = -t^2 e^{-t}$, starting at position $x = 2$.

(a) Determine the position at time t.

(b) Show that the particle never reaches $x = 0$.

——— End of 44th Hundred ———